# CSU/UC Mathematics Diagnostic Testing Project (MDTP) MDTP: Purposes, Recommended Uses, History, Test Development, and Statistical Characteristics 

A Validity and Reliability Study of MDTP Use at the University of California, California State, and Community College Systems

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## Introduction

This document summarizes the purposes, recommended uses, history, development, and statistical characteristics of the tests developed by the California Mathematics Diagnostic Testing Project. The first few sections may help prospective and experienced users find appropriate uses of the tests. The detailed statistical descriptions may be of value to reviewers and institutional researchers. Sample copies of the tests are available for review upon request by qualified users.

## Purpose and Uses of MDTP Tests

The MDTP tests are designed to measure student readiness for a broad range of mathematics courses. More importantly, the tests were also developed to provide students and teachers with diagnostic information about student preparedness. This information can help students identify specific areas where additional study or review is needed. It can help teachers identify topics and skills that need more attention in courses. Institutions may use test results for counseling and assessment, placement, and planning of instruction. The MDTP tests are diagnostic, not comprehensive; they should not be used as final exams.

Schools should help students interpret their individual test results in ways that best meet the students' needs. In particular, each student who takes an MDTP test should be provided with a report not only indicating the student's score but also identifying those topics in which more work is needed. MDTP provides these reports to some users and offers computer software that generates such reports to others. Students whose test results indicate readiness for a mathematics course should be encouraged to pursue their mathematics education. They should also be cautioned that success in a course requires good study habits and persistence. Students whose test results indicate inadequate preparation for a particular course should be advised to develop readiness for that course either by individual study or enrollment in a course that will prepare them for it. They should not be counseled out of mathematics.

Since MDTP tests are designed to measure readiness for further mathematics study, no scaling of scores is provided. Raw scores are used for reporting to students and, when used as part of a placement procedure, for providing one reliable indication of the extent to which a student's current mathematical proficiency matches the skills and knowledge needed for success in a course. Other indicators should also be used. Statistical analysis of test performance and subsequent course performance is a prerequisite to valid uses of test results for placement purposes. This analysis must be undertaken at each school and for each course to determine cut scores and counseling ranges. Moreover, the analysis should continue as long as the tests are used, as they should for any other matriculation assessment instrument.

Another consequence of the purpose of MDTP tests being to measure readiness is that they are criterion-referenced rather than norm-referenced. In Measurement and Evaluation in Psychology and Education 5th ed. (New York: Macmillan, 1991, 194), Thorndike, Cunningham, Thorndike, \& Hagen describe criterion-referenced testing as "a form of evaluation that reports results in terms of what a student can do rather than how the student compares to others." They continue that criterion-referenced tests are used to make absolute, not relative, decisions about whether a student has learned specific content. To avoid any tendency to compare student work to performance of larger
populations, MDTP does not provide data about overall test averages to any of its users. Indeed, these data are not even collected.

Diagnostic test results for groups of students should be provided to mathematics faculty. They can use this information to work with individual students and to identify desirable changes in content or emphasis in their curriculum. Access to test results can also help mathematics faculty monitor the effectiveness of the student placement process.

## Brief Description of MDTP Tests

MDTP tests are offered to California Community Colleges at four levels. The Algebra Readiness Tests measure readiness for a first course in algebra. The Elementary Algebra Tests measure readiness for a second year algebra course. The Intermediate Algebra Tests measure readiness for courses that have second year algebra as a prerequisite. The Precalculus Tests, which measure readiness for calculus, are the most advanced tests offered by MDTP. More detailed descriptions of the topics covered on each MDTP test are included in the Test Forms section of this Manual.

At least two test forms are available at each of the four levels. The more recent forms place more emphasis on conceptual understanding and almost all allow students to use calculators. These changes are consistent with the changing instructional practices recommended by the 1989 NCTM Standards. Allowing or requiring calculator use by students taking MDTP tests also reflects the increasing number of students using calculators.

## Project History

The Mathematics Diagnostic Testing Project was formed as a joint project of The California State University and the University of California in 1977. The charge to the workgroup included determining mathematics areas in which competency was necessary for success in certain mathematics courses and developing diagnostic tests over these areas. By 1986 a series of four tests had been released to California schools.

The tests were originally developed by a workgroup with the financial support of the two California public university systems. In recent years, test development activities of this workgroup have also been supported by fees from California Community Colleges which use MDTP tests. The workgroup members include mathematics faculty from the California public universities, community colleges, and high schools, and university science faculty. The workgroup is assisted in its test development by the consulting services of an expert in mathematics testing from the Educational Testing Service.

Every test that is released by MDTP is first field-tested, revised, and field-tested and revised again, if necessary. One essential criterion to be satisfied by each test is that its topics and items are necessary for success in subsequent mathematics courses. The face validity of the tests' content is evidenced by the composition of the workgroup and by the widespread acceptance and use of the tests throughout California. Every campus of the University of California, approximately two-fifths of the campuses of The California State University, and approximately three-fifths of the campuses of the California Community Colleges use at least one of the MDTP tests as part of their orientation process for entering students. In addition more than 3,300 high school and middle school teachers chose to administer MDTP tests to their students in 1993-1994.

Since 1982 MDTP has offered its tests and scoring services to California high schools. Individual diagnostic reports are provided for students as well as detailed item analyses and summary reports for teachers. The student reports indicate areas in which students did well and those areas in which the test results suggest a need for further study in order to be prepared for future coursework. The summary reports have been used by teachers to help identify areas of the curriculum that seem to be working well and other areas or topics where changes may be needed. Software that scores the tests and produces individual student diagnostic reports is available for use in California postsecondary schools and other educational institutions.

## Field-testing and Test Development Criteria

The critical criterion that each MDTP test must meet is that knowledge of the content of the topics and ability to answer the test's questions are prerequisites for success in subsequent mathematics courses. Evidence that these criteria are satisfied comes from the acceptance of the tests by hundreds of teachers throughout California. Further evidence comes from the data collected during extensive field-testing of the tests. During the final stage of field-testing, the form is administered to students near the beginning of a course so that their test scores can be compared with measures of their performance at the end of the course. For example, the Precalculus Test has been administered at the start of a college calculus course and scores were compared with students' grades in the course. For another example, the Algebra Readiness test has been given to students during the first few weeks of an elementary algebra course and their test scores were compared to both teacher rating of the students' readiness for the next mathematics course at the end of the elementary algebra course and to the students' performance on MDTP's Elementary Algebra Diagnostic Test at the end of the elementary algebra course.

A part of the test development process is an extensive review of item statistics to ensure that each item tests appropriate knowledge and skill, that each item discriminates reasonably well between stronger and weaker students, and that the difficulty levels of the items are not too widespread. The R-Biserial is used as a measure of the consistency of an item with the rest of the test. Content validity of each item is also reviewed using the correlation of item student performance on the item with performance measures at the end of the course. Item discrimination is reviewed in two ways: one is by comparing the overall test performance of the subpopulations who choose each available response (including no response), the other is by comparing the performance of each of the five quintiles of students based on total test score with the performance of the other quintiles.

As new forms of tests are developed, efforts are made to ensure that these forms test the same knowledge domains and tasks at approximately the same level of difficulty. This is done partly by designing the new forms to meet the same predictive validity criteria as the older forms, with test developers maintaining their goal of developing tests of knowledge and skills critical for success in later mathematics courses. The test developers make only minor adjustments to test specifications, as can be seen in the lists of topics on each form later in this Manual. Equating methods are used to compare old and new test forms to ensure that there is no large change in overall test difficulty.

## Test Forms

Several forms of each test have been developed. Some of these forms have only been used for field-testing, others have been released and later retired, while others are still available to universities, colleges, or high schools. In response to some community college requests, Braille versions of all of the 1986 released forms are available. All of the forms of each test have been equated and are sufficiently similar that the history of their statistical characteristics is relevant to understanding the current versions. Every released form differs minimally from a field-test form whose analysis justified the release of the test with very minor changes.

MDTP develops alternate forms of its tests for a variety of reasons. The more recent forms reflect some of the changes in emphasis and approach that are taking place in mathematics education in the United States. These changes are based in part upon the recommendations of the 1989 NCTM Curriculum and Evaluation Standards, in part upon the increasing understanding of ways in which people learn mathematics, and in part upon the ever-growing availability of technology such as calculators which are easy to use and offer students opportunities to see and manipulate more mathematical objects. There is more emphasis in later tests on conceptual understanding than there was on earlier tests. The more recent MDTP tests allow or require students to use calculators in place of early versions which did not allow calculators. Currently, MDTP is trying to find ways to incorporate graphing calculators into assessement of mathematical knowledge and proficiency.

Calculator use may have somewhat unanticipated effects on student performance. [See Brenda H. Loyd, "Mathematics Test Performance: The Effects of Item Type and Calculator Use," Applied Measurement in Education 4 (1991): 11-22.] Users of calculator allowed tests need to be aware of this when developing placement recommendations. It is recommended that student use of calculators on tests be monitored to determine what effects, if any, that use has on future course performance. Colleges have the option, depending on their practices, of using tests that forbid, allow, or require calculators. When changing, care must taken to monitor effects of calculator use on predictive validity of test results. Experience has shown that unexpected effects may occur-these may related to calculator use or to changing instructional practices.

The names of the test forms are constructed by using the first two letters for the abbreviation of the test, the next two digits for the number of items on the test form, the next letter to indicate the version of the test, and the final two digits to indicate the year in which the form was field-tested or released. For example, PC60C86 is one of the forms of the 60 item Precalculus test that was released in 1986.

The remainder of this section consists of tables listing the test forms available at the four levels. The tables include for each test form, its name, whether students may or must or cannot use calculators, and the number of items in each topic area. In addition to summarizing the mathematics on the tests, the topic areas allow colleges to give students diagnostic recommendations. In some newer forms, topic areas from previous forms have been combined or separated to provide clearer diagnostic information. After each table, the number of items and suggested minimum times for available forms are given.

Algebra Readiness Tests (Readiness Tests for First Year Algebra)

| Test Form Calculator | AR50/86 Prohibited | AR50/90 Prohibited | AR50X92 Required |
| :---: | :---: | :---: | :---: |
| Integers, their operations and applications | 10 | 11 | 11 |
| Fractions and their applications | 9 | 11 | 9 |
| Decimals, their operations and applications; Percent | 9 | 8 | 7 |
| Exponents and Square Roots; Scientific Notation | 6 | 5 | 5 |
| Simple Equations and Operations with Literal Symbols | 7 | 5 | 7 |
| Geometry | 9 |  |  |
| Measurement of Geometric Objects |  | 5 | 6 |
| Graphical Representation |  | 5 | 5 |

Each form of the Algebra Readiness Test contains 50 items and has a suggested minimum time of 45 minutes.

## Elementary Algebra Tests (Readiness Tests for Second Year Algebra)

| Test Form <br>  <br> Calculator |  | EA50C86 <br> Prohibited |
| :--- | :---: | :---: |
| EA50A90 <br> Allowed |  |  |
| Arithmetic Operations | 7 | 6 |
| Polynomials | 7 | 7 |
| Linear Equations and Inequalities | 9 | 9 |
| Quadratic Equations | 4 | 4 |
| Graphing | 4 | 6 |
| Rational Expressions | 6 | 5 |
| Exponents and Square Roots | 6 | 6 |
| Geometry | 7 | 7 |

Each form of the Elementary Algebra Test contains 50 items and has a suggested minimum time of 45 minutes.

Intermediate Algebra Tests (Readiness Tests for Courses requiring Second Year Algebra)

| Test Form Calculator | IA45C86 Prohibited | $\begin{aligned} & \hline \text { IA45C91 } \\ & \text { Allowed } \end{aligned}$ |
| :---: | :---: | :---: |
| Elementary Operations | 7 | 6 |
| Rational Expressions | 6 | 6 |
| Exponents and Radicals | 8 | 9 |
| Linear Equations and Inequalities | 6 | 6 |
| Quadratic Polynomials, Equations, and Inequalities | 7 | 7 |
| Graphing and Coordinate Geometry | 6 | 6 |
| Logarithms and Functions | 5 | 5 |

Each form of the Intermediate Algebra Test contains 45 items and has a suggested minimum time of 45 minutes.

Precalculus Tests (Readiness Tests for Calculus)

| Test Form <br>  <br> Calculator | PC60C86 <br> Prohibited | PC60Z90 <br> Required | PC60A93 <br> Allowed |
| :--- | :---: | :---: | :---: |
| Rational Expressions and their Graphs | 8 | 8 | 8 |
| Exponents and Radicals | 9 | 8 | 8 |
| Linear Equations and Inequalities; Absolute | 12 | 11 | 12 |
| Values, and their Graphs | 7 | 7 | 7 |
| Polynomials and Polynomial Functions | 6 | 6 | 7 |
| Functions | 11 | 12 | 11 |
| Trigonometry and Geometry | 7 | 8 | 7 |
| Logarithmic and Exponential Functions |  |  |  |

Each of these forms of the Precalculus Test contains 60 items and has a suggested minimum time of 90 minutes. Each form also is available in an abbreviated version containing 40 items with a suggested minimum time of 60 minutes. The topic distributions for these forms are in the following table.

| Test Form Calculator | PC40C86 Prohibited | PC40Z90 <br> Required | $\begin{aligned} & \hline \text { PC40A93 } \\ & \text { Allowed } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Rational Expressions and their Graphs | 4 | 4 | 5 |
| Exponents and Radicals | 5 | 5 | 4 |
| Linear Equations and Inequalities; Absolute Values, and their Graphs | 8 | 7 | 8 |
| Polynomials and Polynomial Functions | 5 | 6 | 5 |
| Functions | 5 | 5 | 5 |
| Trigonometry and Geometry | 8 | 8 | 7 |
| Logarithmic and Exponential Functions | 5 | 5 | 6 |

## Content Validity

Convincing evidence of the face validity of the tests' content comes from the widespread adoption of the tests by university, college, and high school mathematics faculty throughout California. In addition, the topics on the tests have been shown to be necessary for success in specific courses in a standard academic mathematics curriculum. The experience of one California State University campus, which has made extensive use of MDTP tests in placing students in its mathematics courses, provides further evidence of the tests' content validity. That experience is summarized in the next paragraph.

Since 1984 the Mathematics Department at Cal Poly, Pomona, has used MDTP tests for placement into all introductory General Education level and Preparatory Mathematics Program courses except their arithmetic course. The student success rate in those courses has increased dramatically, from an average of $58 \%$ prior to 1984 to an average of more than $72 \%$ in 1989. The definition of success used in the Cal Poly studies is a grade of C or better in contrast to grades of D, F, U, and W. The changes for each course are indicated in the following chart. The Chair of the Cal Poly Mathematics department has written that "The most dramatic changes have been in the elementary school teachers candidates ...The grades in this one course went from a satisfactory pass average of $64.73 \%$ before [the use of MDTP in placement] to a phenomenal $88.19 \%$ satisfactory pass average as of summer quarter 1989. This is where we believe the MDTP tests are
most affecting the future. These students no longer have any cause to be afraid of mathematics because they themselves have succeeded mathematically."

Percentage of Satisfactory Grades
California State Polytechnic University, Pomona


## Predictive Validity Studies

A comparison of the predictive power of the 60 item Precalculus Test with other available predictors was made in the early 1980s. Student performance in the first course in each of the UCLA calculus sequences was predicted far more strongly by the 60 item MDTP Precalculus Test score than by the other predictors. The correlations (r) are listed in Table One.

Table One

Predictor
Physical Sciences
Calculus

| PC60 Test Score | .61 | .49 |
| :--- | :---: | :---: |
| SAT Verbal | .18 | .06 |
| SAT Math | .31 | .28 |
| High School GPA | .23 | .47 |

In 1981 a similar comparative study of the 40 item Precalculus Test was done at the Davis campus of the University of California. The correlations were computed based on totals of 131 students in physical sciences calculus and 168 students in life sciences calculus. The correlations (r) are listed in Table Two.

| Predictor | Table Two <br> Physical Sciences <br> Calculus | Life/Social Sciences <br> Calculus |
| :--- | :---: | :---: |
| PC40 Test Score | .54 | .39 |
| SAT Verbal | .12 | .20 |
| SAT Math | .44 | .33 |
| High School GPA | .35 | .36 |
| CEEB Mathematics 1 | .35 | .32 |

There have been a large number of studies of the correlation between various versions of the Precalculus Test and performance in college calculus courses. That performance is measured by final grades except for the Santa Barbara City College study where midterm grades were used. The tests were not used for placement in the first three studies listed below, but were used as part of the placement process in all of the others.

Table Three

| Released <br> Form | Field- <br> Tested <br> Form | School |  |  |
| :--- | :--- | :--- | ---: | ---: |
| NC60/80 | PC60/79 | Correlation |  |  |
| PC60/80 | PC60/79 | Sacramento State | 459 | .61 |
| PC60/80 | PC60/79 | CSU LA | 311 | .55 |
| PC60/80 | PC60/80 | Sacramento State | 111 | .59 |
| PC60C86 | PC60A84 | Sacramento State | 462 | .36 |
| PC60C86 | PC60B84 | Sacramento State | 118 | .52 |
| PC60C86 | PC60C84 | Sacramento State | 143 | .30 |
| PC60C86 | PC60C86 | Santa Barbara City College | 127 | .47 |
| PC60Z90 | PC60E88 | Sacramento State | 64 | .38 |
| PC60A933 | PC60F91 | UCLA | 321 | .39 |
| PC40C86 | PC40A85 | UC Davis | 136 | .48 |
| PC40C86 | PC40A85 | Cal Poly SLO | 163 | .51 |
| PC40C86 | PC40B85 | UC Davis | 106 | .34 |
| PC40C86 | PC40B85 | Cal Poly SLO | 161 | .33 |
| PC40C86 | PC40C85 | UC Davis | 107 | .36 |
| PC40C86 | PC40C85 | Cal Poly SLO | 124 | .54 |

Reports from a number of studies of the correlation between scores on the Intermediate Algebra Test and final course grades in college precalculus courses are available. They are listed in Table Four. In all cases, the test scores were a factor in student placement into the course. The next to the last entry in the table is based on a 1990 study of the correlation with midterm grades in a variety of courses. The last entry presents a correlation with administration of MDTP's 40 item precalculus diagnostic test near the end of mathematics analysis courses.

## Table Four

| Released <br> Form | Field-Tested | Form |  | School |
| :--- | :--- | :--- | ---: | ---: |

Table Five lists the reported correlations between scores on the MDTP Elementary Algebra Test and subsequent performance in college intermediate algebra courses. That performance is measured by final grades except for the next to the last entry where midterm grades were used. One large analysis was done using 1,073 California high school students showing that their scores on EA50D88 had a correlation of 0.50 with their subsequent performance in high school intermediate algebra courses.

Table Five

| Released <br> Form | Field-Tested |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
|  | Form | School | N | Correlation |
|  | EA50/81 | CSU Chico | 291 | .38 |
|  | EA50/81 | Sacramento State | 328 | .41 |
|  | EA50A84 | Sacramento State | 95 | .43 |
|  | EA50B84 | Sacramento State | 71 | .62 |
| EA50C86 | EA50A85 | Sacramento State | 394 | .34 |
| EA50C86 | EA50B85 | Sacramento State | 399 | .27 |
| EA50C86 | EA50C85 | Sacramento State | .38 |  |
| EA50C86 | EA50C86 | Santa Barbara City College | 472 | .40 |
| EA50A90 | EA50D88 | California High Schools | 1073 | .50 |

The Algebra Readiness Test was developed starting in 1984. Originally it was called the Prealgebra Test, so that the early test forms were abbreviated PA. Correlations with end of course performance in subsequent elementary algebra courses are available for a broad sample of California high schools and three community colleges. Test scores were not used in placing students in these courses except for the study of Santa Barbara City College students. The courses at Chabot Community College were elementary algebra, first quarter intermediate algebra, and arithmetic, in the order reported in the table. The midterm grades in the elementary algebra course were used in the Santa Barbara City College study.

Table Six

| Released <br> Form | Field- <br> Tested <br> Form | School | N | Correlation |
| :--- | :--- | :--- | ---: | ---: |
|  | PA50/84 | California High Schools | 2000 | .42 |
| PA50/84 | Sacramento City College | 199 | .38 |  |
| AR50/86 | PA50/85 | California High Schools | 2000 | .43 |
| AR50/86 | PA50/85 | Sacramento City College | 275 | .38 |
| AR50/86 | PA50/85 | Urban high school district | 816 | .49 |
| AR50/86 | PA50/85 | Chabot College | 403 | .35 |
| AR50/86 | PA50/85 | Chabot College | 354 | .46 |
| AR50/86 | PA50/85 | Chabot College | 333 | .39 |
| AR50/86 | AR50/86 | Santa Barbara City College | 451 | .43 |
| AR50/90 | AR50D89 | California High Schools | 372 | .39 |
| AR50X92 | AR50U91 | California High Schools | 412 | .52 |

## Reliability Studies

Kuder-Richardson
The Kuder-Richardson 20 estimate of inter-item correlations was computed in most of the analyses done during MDTP test development and some subsequent studies. Samples of the values computed are listed in this section.

Table Seven lists computed Kuder-Richardson 20 reliability coefficients for the 60 item Precalculus Test forms.

| Released | Field-Tested | Table Seven |  |  |
| :--- | :--- | :--- | ---: | ---: |
| Form | Form | School | N | KR 20 |
| PC60/80 | PC60/80 | Sacramento State | 643 | 0.91 |
|  | PC60A84 | UC Berkeley | 194 | 0.89 |
|  | PC60A84 | Sacramento State | 202 | 0.90 |
|  | PC60B84 | UC Berkeley | 173 | 0.91 |
|  | PC60B84 | Sacramento State | 204 | 0.88 |
|  | PC60C84 | UC Berkeley | 219 | 0.90 |
|  | PC60C84 | Sacramento State | 196 | 0.91 |
| PC60A86 | PC60A86 | California Colleges | 433 | 0.91 |
| PC60C86 | PC60C86 | California Colleges | 871 | 0.90 |
| PC60C86 | PC60C86 | Santa Barbara City College | 237 | 0.91 |
| PC60Z90 | PC60E88 | Sacramento State | 820 | 0.95 |
| PC60A93 | PC60F91 | UCLA | 1499 | 0.92 |

Table Eight lists computed Kuder-Richardson 20 reliability coefficients for the 40 item Precalculus Test forms.

Table Eight

| Released | Field-Tested | Table Eight |  |  |
| :--- | :--- | :--- | ---: | ---: |
| Form | Form | School | N | KR 20 |
| PC40/80 | PC40/80 | UCSD | 1304 | 0.87 |
|  | PC40A85 | CSU Northridge | 90 | 0.84 |
|  | PC40A85 | UC Davis | 259 | 0.86 |
|  | PC40A85 | Cal Poly SLO | 207 | 0.88 |
|  | PC40B85 | CSU Northridge | 85 | 0.85 |
|  | PC40B85 | UC Davis | 253 | 0.88 |
|  | PC40B85 | Cal Poly SLO | 200 | 0.87 |
| PC40C86 | PC40C85 | CSU Northridge | 82 | 0.85 |
| PC40C86 | PC40C85 | UC Davis | 253 | 0.85 |
| PC40C86 | PC40C85 | Cal Poly SLO | 167 | 0.89 |
| PC40C86 | PC40C86 | California Colleges | 313 | 0.76 |

Table Nine lists computed Kuder-Richardson 20 reliability coefficients for the Intermediate Algebra Test forms.

Table Nine

| Released <br> Form | Field-Tested |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
|  | Form | School |  | N | KR 20

Table Ten lists computed Kuder-Richardson 20 reliability coefficients for the Elementary Algebra Test forms.

Table Ten

| Released <br> Form | Field-Tested |  |  |  |
| :--- | :--- | :--- | ---: | ---: |
|  | Form | School | KR 20 |  |
|  | EA50/81 | CSU Chico | 303 | 0.85 |
|  | EA50/81 | Sacramento State | 415 | 0.91 |
|  | EA50/82 | Sacramento State | 843 | 0.89 |
|  | EA50A84 | Sacramento State | 202 | 0.91 |
|  | EA50B84 | Sacramento State | 181 | 0.91 |
|  | EA50C84 | Sacramento State | 112 | 0.84 |
|  | EA50A85 | Sacramento State | 554 | 0.91 |
|  | EA50B85 | Sacramento State | 566 | 0.90 |
| EA50C86 | EA50C85 | Sacramento State | 570 | 0.90 |
|  | EA50A86 | California Colleges | 1044 | 0.87 |
| EA50C86 | EA50C86 | California Colleges | 478 | 0.87 |
| EA50C86 | EA50C86 | Santa Barbara City College | 1267 | 0.90 |
| EA50A90 | EA50D88 | California High Schools | 1073 | 0.87 |

Table Eleven lists computed Kuder-Richardson 20 reliability coefficients for the Algebra Readiness Test forms. Since the Algebra Readiness Test was called the Prealgebra Test until 1986, the first two forms are called PA50/84 and PA50/85.

Table Eleven

| Released <br> Form | Field- <br> Tested <br> Form | School | N | KR 20 |
| :--- | :--- | :--- | ---: | ---: |
|  | PA50/84 | California High Schools | 2000 | 0.82 |
|  | PA50/84 | Sacramento City College | 368 | 0.84 |
| AR50/86 | PA50/85 | California High Schools | 2000 | 0.85 |
| AR50/86 | PA50/85 | Sacramento City College | 478 | 0.86 |
| AR50/86 | AR50/86 | Suburban District | 1853 | 0.83 |
| AR50/86 | AR50/86 | Santa Barbara City College | 1601 | 0.91 |
| AR50/90 | AR50D89 | California High Schools | 372 | 0.88 |
| AR50X92 | AR50U91 | California High Schools | 785 | 0.90 |

## Test-Retest

In 1990 test-retest studies of MDTP tests were conducted at Santa Barbara City College. Students were retested after an interval of two weeks to six months. Only those students who reported no intervening mathematics experience were included in the computation of the correlations listed in Table Twelve.

Table Twelve

| Test | N | Correlation |
| :--- | ---: | ---: |
| AR50/86 | 453 | .87 |
| EA50C86 | 290 | .81 |
| IA45C86 | 464 | .67 |
| PC60C86 | 62 | .76 |

## Standard Error of Measurement

An estimate of the Standard Error of Measurement was computed in most of the developmental studies done for MDTP tests.

Almost all of the Precalculus Test studies were done with students at various campuses of The California State University and the University of California. The only exception is the last reported study of PC60C86, which was done at Santa Barbara City College. For the 60 item forms, the computed standard errors of measurement were always in the range from 3.13 to 3.37 .

## Table Thirteen

| Released <br> Form | Field- <br> Tested | N | S. E. M. |
| :--- | :--- | ---: | ---: |
| Form |  |  |  |$\quad$|  |  |  |
| :--- | :--- | ---: |
| PC60/80 | PC60/80 | 1181 |

The standard errors of measurement computed on the 40 item Precalculus Test forms ranged from 2.62 to 2.74 . When the colleges participating in a study are known, they are named in the third column of Table Fourteen.

| Released <br> Form | Field- <br> Tested <br> Form | School |  |  |
| :--- | :--- | :--- | ---: | ---: |
|  |  | N | S.E.M. |  |
| PC40/80 | PC40/80 | UCSD |  |  |
|  | PC40A85 | CSUN, UCD, SLO | 1304 | 2.62 |
|  | PC40B85 | CSUN, UCD, SLO | 536 | 2.62 |
| PC40C86 | PC40C85 | CSUN, UCD, SLO | 502 | 2.74 |
| PC40C86 | PC40C86 |  | 313 | 2.74 |
|  |  |  | 2.73 |  |

The standard errors of measurement computed on the 45 item Intermediate Algebra Test forms ranged from 2.74 to 2.90 . When the colleges participating in a study are known, they are named in the third column of the following table.

Table Fifteen

| Released <br> Form | Field- <br> Tested <br> Form | School | N | S. E. M. |
| :--- | :--- | :--- | ---: | ---: |
|  |  |  | 938 | 2.85 |
|  | IA45/82 | Sacramento State | 511 | 2.80 |
|  | IA45A85 | CSUN, Sac State, SLO | 1482 | 2.84 |
|  | IA45A86 |  | 194 | 2.81 |
|  | IA45B84 | UCLA | 512 | 2.80 |
|  | IA45B85 | CSUN, Sac State, SLO | 146 | 2.86 |
|  | IA45C84 | UCLA | 477 | 2.86 |
| IA45C86 | IA45C85 | CSUN, Sac State, SLO | 1481 | 2.90 |
| IA45C86 | IA45C86 |  | 769 | 2.86 |
| IA45C86 | IA45C86 | Santa Barbara City College | 525 | 2.74 |
| IA45C91 | IA45D90 | California High Schools |  |  |

The standard errors of measurement computed on Elementary Algebra Test forms ranged from 2.88 to 3.12 . When the schools participating in a study are known, they are named in the third column of Table Sixteen.

Table Sixteen

| Released <br> Form | Field- <br> Tested | School | N | S. E. M. |
| :--- | :--- | :--- | ---: | ---: |
|  | Form |  |  |  |

For the Algebra Readiness Test, all of the estimated standard errors of measurement were in the range from 2.95 to 3.06 . Since the Algebra Readiness Test was called the Prealgebra Test until 1986, its first two forms are called PA50/84 and PA50/85. The standard errors of measurement are listed with population descriptions and the number of students in each sample in Table Seventeen.

| Table Seventeen |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Released Form | Field- | School | N | S. E. M. |
|  | Tested |  |  |  |
|  | Form |  |  |  |
|  | PA50/84 | High School | 2000 | 3.02 |
|  | PA50/84 | Sacramento City College | 368 | 2.99 |
| AR50/86 | PA50/85 | High School | 2000 | 2.95 |
| AR50/86 | PA50/85 | Sacramento City College | 478 | 2.95 |
| AR50/86 | PA50/85 | Urban district | 1304 | 2.95 |
| AR50/86 | AR50/86 | Suburban district | 1853 | 3.06 |
| AR50/86 | AR50/86 | Santa Barbara City College | 1601 | 2.95 |
| AR50/90 | AR50D89 | California High Schools | 372 | 3.03 |
| AR50X92 | AR50U91 | California High Schools | 785 | 3.03 |

A 1990 study at Santa Barbara City College computed standard errors of measurement for the MDTP tests for intervals of raw scores. The results of that study are reported in the following tables.

An analysis of 237 Precalculus Test scores provided the estimates of standard errors of measurement for the score intervals listed in Table Eighteen. The test form was PC60C86.

Table Eighteen

| Score | N | S. E. M . |
| :---: | ---: | ---: |
| $0-5$ | 0 | $* * *$ |
| $6-10$ | 2 | 1.42 |
| $11-15$ | 5 | 2.62 |
| $16-20$ | 10 | 3.10 |
| $21-25$ | 17 | 3.38 |
| $26-30$ | 21 | 3.44 |
| $31-35$ | 39 | 3.46 |
| $36-40$ | 41 | 3.36 |
| $41-45$ | 33 | 3.19 |
| $46-50$ | 35 | 2.92 |
| $51-55$ | 24 | 2.40 |
| $56-60$ | 10 | 1.62 |

An analysis of 769 Intermediate Algebra Test scores provided the estimates of standard errors of measurement for the score intervals listed in Table Nineteen. The test form was IA45C86.

Table Nineteen

| Score | N | S. E. M. |
| ---: | ---: | ---: |
| $0-5$ | 8 | 1.73 |
| $6-10$ | 48 | 2.48 |
| $11-15$ | 128 | 2.86 |
| $16-20$ | 173 | 2.94 |
| $21-25$ | 170 | 2.96 |
| $26-30$ | 124 | 2.85 |
| $31-35$ | 70 | 2.66 |
| $36-40$ | 40 | 2.26 |
| $41-45$ | 8 | 1.56 |

An analysis of 1267 Elementary Algebra Test scores provided the estimates of standard errors of measurement for the score intervals listed in Table Twenty. The test form was EA50C86.

| Table Twenty |  |  |
| :---: | ---: | ---: |
| Score | N | S. E. M. |
| $0-5$ | 8 | 1.73 |
| $6-10$ | 78 | 2.50 |
| $11-15$ | 191 | 2.90 |
| $16-20$ | 254 | 3.12 |
| $21-25$ | 207 | 3.22 |
| $26-30$ | 213 | 3.22 |
| $31-35$ | 152 | 3.05 |
| $36-40$ | 101 | 2.86 |
| $41-45$ | 50 | 2.37 |
| $46-50$ | 13 | 1.28 |

An analysis of 1601 Algebra Readiness Test scores provided the estimates of standard errors of measurement for the score intervals listed in Table Twenty-one. The test form was AR50/86.

Table Twenty-one

| Score |  | N |
| ---: | ---: | ---: |
| S. E. M. |  |  |
| $0-5$ | 27 | 1.47 |
| $6-10$ | 62 | 2.40 |
| $11-15$ | 155 | 2.86 |
| $16-20$ | 246 | 3.03 |
| $21-25$ | 267 | 3.09 |
| $26-30$ | 296 | 3.06 |
| $31-35$ | 272 | 2.95 |
| $36-40$ | 180 | 2.73 |
| $41-45$ | 80 | 2.31 |
| $46-50$ | 16 | 1.68 |

## Bias Issues

Four studies that address possible MDTP test bias based upon gender or ethnicity have been completed. Two of these studies investigated predictive validity of the tests, one using school district data and another using data from a community college district. A third study, using data from another community college district, investigated possible bias of individual test items. Finally, a California State University campus study explored the effects on different ethnic groups of using MDTP tests for placement.

The first study of predictive validity was done in a large urban California school district with the understanding that the district would not be identified in any dissemination of the results. The correlations of test scores with subsequent course grades in elementary algebra courses are listed in Table Twenty-two.

|  | Table Twenty-two |  |  |
| :--- | :--- | ---: | ---: |
| Test Form | Category | N | Correlation |
| PA50/85 | Male | 397 | .49 |
| PA50/85 | Female | 409 | .50 |
| PA50/85 | White | 367 | .42 |
| PA50/85 | Black | 223 | .53 |
| PA50/85 | Hispanic | 74 | .49 |
| PA50/85 | Asian | 34 | .52 |

The second predictive validity study was done at Santa Barbara City College in 1990. All computed correlations are listed even though several lack significance due to their small sample sizes. Table Twenty-three lists correlations of Precalculus Test scores with midterm calculus course grades. Students who withdrew from the course are included in the population.

|  | Table Twenty-three |  |  |
| :--- | :--- | ---: | ---: |
| Test Form | Category | N | Correlation |
| PC60C86 | Male | 46 | .47 |
| PC60C86 | Female | 20 | .52 |
| PC60C86 | White | 48 | .53 |
| PC60C86 | Black | 1 | n/a |
| PC60C86 | Hispanic | 5 | .60 |
| PC60C86 | Asian | 11 | .22 |

Table Twenty-four lists correlations of Intermediate Algebra Test scores with midterm grades in a variety of courses, including college algebra, trigonometry, and statistics. Students who withdrew from a course are included in the population.

|  | Table Twenty-four |  |  |
| :--- | :--- | ---: | ---: |
| Test Form | Category | Correlation |  |
| IA45C86 | Male | 115 | .34 |
| IA45C86 | Female | 78 | .46 |
| IA45C86 | White | 137 | .35 |
| IA45C86 | Black | 3 | .81 |
| IA45C86 | Hispanic | 27 | .22 |
| IA45C86 | Asian | 15 | .61 |

The next table lists correlations of Elementary Algebra Test scores with midterm grades in intermediate algebra classes. Students who withdrew are included in the population.

|  | Table Twenty-five |  |  |
| :--- | :--- | ---: | ---: |
| Test Form | Category | N | Correlation |
| EA50C86 | Male | 234 | .35 |
| EA50C86 | Female | 271 | .33 |
| EA50C86 | White | 390 | .32 |
| EA50C86 | Black | 11 | .35 |
| EA50C86 | Hispanic | 69 | .32 |
| EA50C86 | Asian | 15 | .36 |

Table Twenty-six lists correlations of Algebra Readiness Test scores with midterm grades in elementary algebra classes. Students who withdrew are included in the population.

Table Twenty-six

| Test Form | Category | N | Correlation |
| :--- | :--- | ---: | ---: |
| AR50/86 | Male | 229 | .39 |
| AR50/86 | Female | 263 | .37 |
| AR50/86 | White | 367 | .35 |
| AR50/86 | Black | 16 | .73 |
| AR50/86 | Hispanic | 77 | .30 |
| AR50/86 | Asian | 12 | .45 |

Possible test bias was also studied using 1990-1991 data from approximately 13,500 students in the Los Angeles Community College District. The district, which has a diverse student population, administers tests in roughly the same proportion as tests are administered throughout the California community colleges. The four tests studied were the Algebra Readiness Test (AR50/86), the Elementary Algebra Diagnostic Test (EA50C86), the Intermediate Algebra Diagnostic Test (IA45C86), and the Precalculus Diagnostic Test (PC40C86). The ethnic classifications used in this study were Asian, Black, Hispanic, and White. The numbers of students reporting other ethnicities were too small to provide reliable data.

The Mantel-Haenszel Differential Item Functioning analysis was used to identify items which, based upon the responses of the various groups, were candidates for showing test bias. No items were such candidates for gender bias on any of the tests. The numbers of candidates for showing test bias on each of the tests except the Algebra Readiness Test were small enough that reviewers were not concerned about test bias on those tests. The only pattern of possible bias that concerned reviewers was that involving Asian students on the Algebra Readiness Test.

Twenty-one of the fifty items were candidates for showing possible bias comparing Asian students to at least one of the other three ethnic groups according to the Mantel-Haenszel criteria. Thirteen of these were possibly easier for Asian students than for one or more of the other groups while eight were possibly harder for Asian students. Of the nine items that appear as candidates in all three ethnic comparisons involving Asians, four appeared consistently more difficult for Asian students and five easier for Asian students. All four of the more difficult candidates are verbally loaded, all five of the easier candidates require computation and no interpretation.

To more fully explore possible test bias, MDTP convened a four member panel to consider the possible bias against Asian students of some items on the Algebra Readiness Test. Each of the panelists had experience teaching mathematics. The ethnic backgrounds of the panelists included Chinese, Japanese, Korean, and Vietnamese. Two of the panelists were women, and two were men. After all the suspect items had been reviewed, the panelists agreed that no questions were deemed biased against Asian students as a group. The panelists further agreed that all the problems were appropriate for a test measuring readiness for a first Algebra course in a California community college. The panelists also noted that linguistic problems of students create difficulties in instruction as well as difficulties for the students. Therefore, they found that measuring linguistic proficiencies is appropriate as part of the assessment component of a community college matriculation process.

The Pomona studies mentioned in the Content Validity section also address the impact of MDTP tests on various student ethnic groups. The Chair's findings are nicely summarized in the following two paragraphs taken from her report.

> We are very concerned about how the MDTP affects all our students, especially our underrepresented students. We work very closely with all our ethnic minorities, disabled students and all other students with special needs to be sure they are tested under whatever conditions are necessary to meet their needs. This guarantees that they, too, will be properly placed into the correct math class.
> In particular, the MDTP testing has significantly boosted our underrepresented students, selfesteem as well as their mathematics ability. Since there are no exceptions to any student satisfying the appropriate prerequisite, both our preparatory and GE-level math classes are now fully integrated. We no longer have a disproportionate number of minority students scoring in the lowest quartile of the grading scale in these or any of our math classes.

## Acknowledgements and Updates

This manual was written by Carol Gerachis and Alfred Manaster. They are both members of the MDTP workgroup. The majority of the analyses of MDTP tests has been carried out by Wally Etterbeek, who is the MDTP statistician as well as a member of the MDTP workgroup. Additional data and analyses in this document were provided to us by Leigh Burstein, William Duxler, Robert Elmore, and Eve Kelemen-Lohnas. We are grateful to them for their cooperation. Results of studies conducted by Merriline Smith, Ward Stewart, William Threlfall, and Bokhee Yoon are also included. We appreciate all of their cooperation. Finally, we appreciate the careful review and insights provided by Mary Anne Anthony and Chancey Jones.

The workgroup would appreciate receiving copies of similar studies involving MDTP tests. Users of the tests who gather such data are asked to send copies to Alfred Manaster, Department of Mathematics 0112, 9500 Gilman Drive, University of California, San Diego, La Jolla, CA 92093-0112. We hope to include results of other studies in subsequent statistical summaries.

